

13 Years of TOPEX/Poseidon Precision Orbit Determination and the 10-fold improvement in expected orbit accuracy

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Launched in the summer of 1992, TOPEX/Poseidon (T/P) was a joint mission between NASA and the Centre National d'Etudes Spatiales (CNES), the French Space Agency, to make precise radar altimeter measurements of the ocean surface. After the remarkably successful 13 years of mapping the ocean surface T/P lost its ability to maneuver and was de-commissioned in January 2006. T/P has revolutionized the study of the Earth's oceans by vastly exceeding pre-launch estimates of surface height accuracy recoverable from radar altimeter measurements. The precision orbit provides the reference frame from which the radar altimeter measurements are made. The expected quality of orbit knowledge had limited the measurement accuracy expectations of past altimeter missions, and still remains a major component in the error budget of all altimeter missions. This paper describes critical improvements made to the T/P orbit time series over the 13 years of precise orbit determination (POD) provided by the NASA GSFC Space Geodesy Laboratory. In this paper we review the POD improvements from the pre-launch T/P expectation of radial orbit accuracy and mission requirement of 13 cm to an expected accuracy of about 1.5 cm with today's latest orbits. The latest orbits with 1.5 cm RMS radial accuracy represent a significant improvement to the 2.0 cm accuracy orbits currently available on the T/P Geophysical Data Record (GDR) altimeter product.

Nomenclature

Subscripts

l spherical harmonic degree

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m spherical harmonic order

Conventions

$\bar{C}_{lm}, \bar{S}_{lm}$ Normalized spherical harmonic coefficients

\bar{P}_{lm} Normalized associated Legendre function

c_d Drag coefficient

GM Gravitational constant, m^3/s^2

K_p Geomagnetic activity index

r radius

R_e Reference radius for gravity model

U Gravity potential

W_{jk} Constraint equation weight between pairs of parameters

A/S Antispoofing

CNES Centre National D'Etudes Spatiales

cpr cycle per revolution

DORIS Doppler orbitography and radiopositioning integrated by satellite

ENSO El Niño Southern Oscillation

GDR Geophysical Data Record

GEM Goddard Earth Model

GGM Grace Gravity Model

GPS Global Positioning System

JGM Joint Gravity Model

LEO Low Earth Orbit

LRA Laser Retroreflector Array

opr one-cycle per revolution

POD Precision orbit determination

POE Precision orbit ephemerides

SLR Satellite Laser Ranging

T/P TOPEX/Poseidon

TDRSS Tracking Data Relay Satellite System

Symbols

λ longitude

ϕ latitude

σ standard deviation

τ Correlation time

I. Introduction

TOPEX/Poseidon (T/P) is the most successful space mission specifically designed for studying the circulation of the world's oceans which cover 70% of the planet. T/P, Jason-1 (the T/P follow-on) and all other satellite altimeter missions use advanced radar altimetry to monitor the changing shape of the sea surface. The sea surface height measurements reveal worldwide patterns of the ocean circulation and help clarify the role of ocean currents in the changing climate of our ocean planet.¹ The sea surface height measurements are made by computing the surface of the ocean with respect to a reference ellipsoid. In order to accomplish this, the radial position of the satellite must be known as accurately as possible.² Oceanography has been radically changed thanks to the global and precise observations from satellite altimetry. The recent NASA review of current Earth observing missions has ranked Jason-1 and T/P at the top for scientific achievement and relevance.³ T/P and Jason-1 scientific results include monitoring of ENSO (El Niño and La Niña) events, accurate determination of global tides, geostrophic currents, and ocean topography, delineation of Rossby waves, and measurement of mean sea level trends. With global warming and the uncertainty of its effect on our changing climate and biosphere, one cannot overestimate the importance of information obtained from altimeter data analysis, especially in combination with other data, such as sea surface temperature. We show in Fig. 1 the strong correlation between mean sea level rise derived from T/P and Jason-1 data, and the rise in sea surface temperature, beginning in 1993.

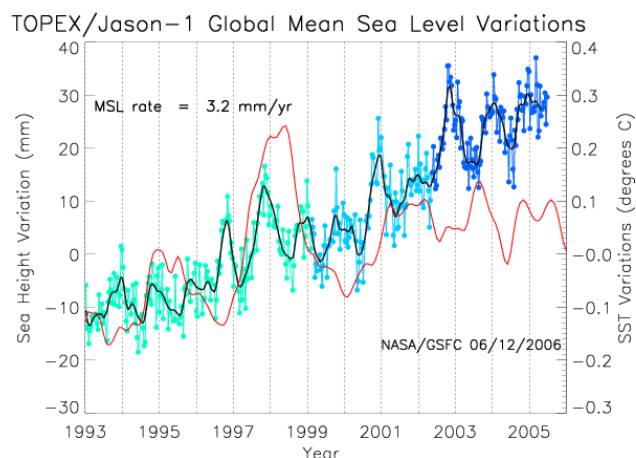


Figure 1. Sea level variations from TOPEX/Poseidon and Jason-1 (1993-2005). The mean sea level rate is 3.2 mm/year, but includes annual and interannual variations such as the 1997-1998 ENSO event.

improved satellite tracking technologies. This paper describes the history of significant improvements to T/P Precision Orbit Determination (POD) which permitted the routine computation of 2 cm orbits at GSFC for use on the T/P Geophysical Data Record (GDR), current advances in T/P POD, and anticipated future work.

Launched on August 10 1992, TOPEX/Poseidon was a joint mission between NASA and the Centre National d'Etudes Spatiales (CNES), the French Space Agency, to make precise radar altimeter measurements of the ocean surface.¹ After 13 years of mapping the ocean surface, T/P lost its ability to maneuver and was de-commissioned in January 2006. The T/P satellite flew in a near-circular, frozen orbit at an average height of 1336 km with an inclination of 66°. It has a period of 112.4 minutes and a ground-track repeat cycle of 9.916 days after completing 127 orbital revolutions. T/P included four tracking systems, which were used routinely for precision orbit determination:^{1,6}

- (1) Satellite laser ranging (SLR).⁷
- (2) DORIS.^{8,9}
- (3) GPS.¹⁰
- (4) The TOPEX altimeter.

The altimeter data from T/P constitute an additional data type which can be used for orbit determination and validation. In addition to these four tracking systems, T/P carried a high gain antenna for communication with TDRSS. Although the TDRSS data were not routinely used for T/P precision orbit determination, they have been used to improve POD for the TDRSS satellites and other LEO satellites tracked through TDRSS by connecting these satellites to the precise orbits available from SLR, DORIS, or GPS on TOPEX/Poseidon.^{11,12}

One of the achievements of the GPS system on T/P was to permit the calculation of orbits using a reduced-dynamic technique.¹³ The intercomparison of the reduced-dynamic orbits available from GPS, and the dynamic orbits available from SLR and DORIS provided a valuable means to identify and eventually reduce force model errors, such as geographically correlated orbit error from gravity field mismodeling, and stationary errors from dynamic ocean tide mismodeling.^{5,14} The GPS Monarch receiver could only track GPS satellites through six channels and operated in dual-frequency mode only while A/S (antispoofing)

T/P has revolutionized the study of the Earth's oceans by vastly exceeding pre-launch estimates of surface height accuracy recoverable from radar altimeter measurements. The expected quality of orbit knowledge had limited the measurement accuracy expectations of past altimeter missions, and still remains a major component in the error budget of all altimeter missions including T/P (see Table 1). The T/P mission has allowed a breakthrough in our understanding of large-scale variability of sea surface heights as well as of the global mean sea level. The T/P mission represents the first time that the radial component of an altimeter orbit has been routinely computed with an accuracy approaching 2 cm. The giant leap forward in Low Earth Orbiting (LEO) satellite orbit accuracy which began with T/P was due to improvements in modeling forces acting on the satellite, and especially the gravity field, improvements in the terrestrial reference frame, and

Table 1. TOPEX/Poseidon Altimeter Range Error Budget

Component	Error (cm)	Source
Altimeter		
w/out biases	3.7	(Carlisle, 1991) ⁴
biases	3.0	(Carlisle, 1991) ⁴
radial orbit	2.5	(Marshall et al., 1995) ⁵
Total RSS	5.4	

was off. This limited the use of the GPS data for precise orbit applications to approximately the first 16 months in orbit or through January 1994. Nonetheless, we note even the T/P GPS A/S data have been used to produce quick-look orbits with a latency of several days.¹⁵ The precise orbits that are used for the geophysical data records (GDR's) rely on the processing and analysis of SLR, DORIS, and altimeter crossover data.

Although there is no measure of absolute orbit accuracy, an ensemble of tests including data residual analysis, orbit consistency between overlapping data periods, and direct orbit comparison have been used to evaluate 1 cm accuracies and are used in this study.^{16–18} Improvement of the T/P orbit has led to better consistency between the T/P and Jason-1 (T/P follow-on mission) datasets.¹⁹

II. Orbit Error Budget

At the time of launch in 1992, the T/P altimeter mission had one of the most stringent orbit determination requirements ever imposed on a satellite mission: 13 cm RMS (1σ) radial accuracy. Today, the latest T/P orbits computed at GSFC have improved by nearly a factor of ten compared to these pre-launch requirements (see Table 2). Not only have the GDR orbits allowed a breakthrough in our understanding of large scale variability of sea surface heights, the most recent improvements in T/P orbit accuracy have led to significant improvements in consistency between the T/P and Jason-1 datasets, and promise further improvement in modeling the altimeter range, especially in the sea state bias correction.¹⁹

Table 2. TOPEX/Poseidon Radio Orbit Error Budget (cm)

Orbit Error Component	Mission Requirement 1992	1st Generation POE	2nd Generation POE (GDR)	Latest Generation
Gravity	10	2	1	< 1
Radiative Forces	6	2	< 2	1
Atmospheric Drag	3	1	< 1	<1
Earth & Ocean Tides	3	2	1	< 1
Station Location	2	1	< 1	< 1
Other	2	1	1	< 1
Total	13	3-4	2-3	1.5

III. Achieving the GDR Orbit

We show in Fig. 2 the uniform high degree of consistency and accuracy of the GDR orbits with three orbit quality indices: the SLR tracking data residuals (mismatch between the observed and computed values), the radial orbit differences with independent solutions computed at the University of Texas/Center for Space Research (CSR) Analysis center, and the radial orbit differences over overlapping data periods between adjacent arcs. The analysis of these and other orbit quality indices indicate the T/P GDR orbits have a radial accuracy approaching 2 cm.⁵ We detail the POD advances leading to the GDR orbits, with a more detailed description of the three POD models that contributed to improvements in GDR orbit accuracy: the surface force modeling, the gravity model and the tide models.

III.A. A Short History

Isaac Newton once remarked: "If I have seen a little further it is by standing on the shoulders of Giants." The groundwork and possibility for achieving any such advances in POD modeling was laid over the course of a 10-year multi-institutional effort initiated in 1982 to improve the gravity field and the satellite force models in anticipation of the T/P mission.²⁰

The 10-year pre-launch analysis effort produced the Joint Gravity Model-1 (JGM-1),²¹ the surface force macromodel,^{22,23} the T/P Laser Retroreflector Array (LRA) model, among others. It was however, by far the early advances in gravity field modeling culminating in the pre-launch JGM-1 model and its extensive

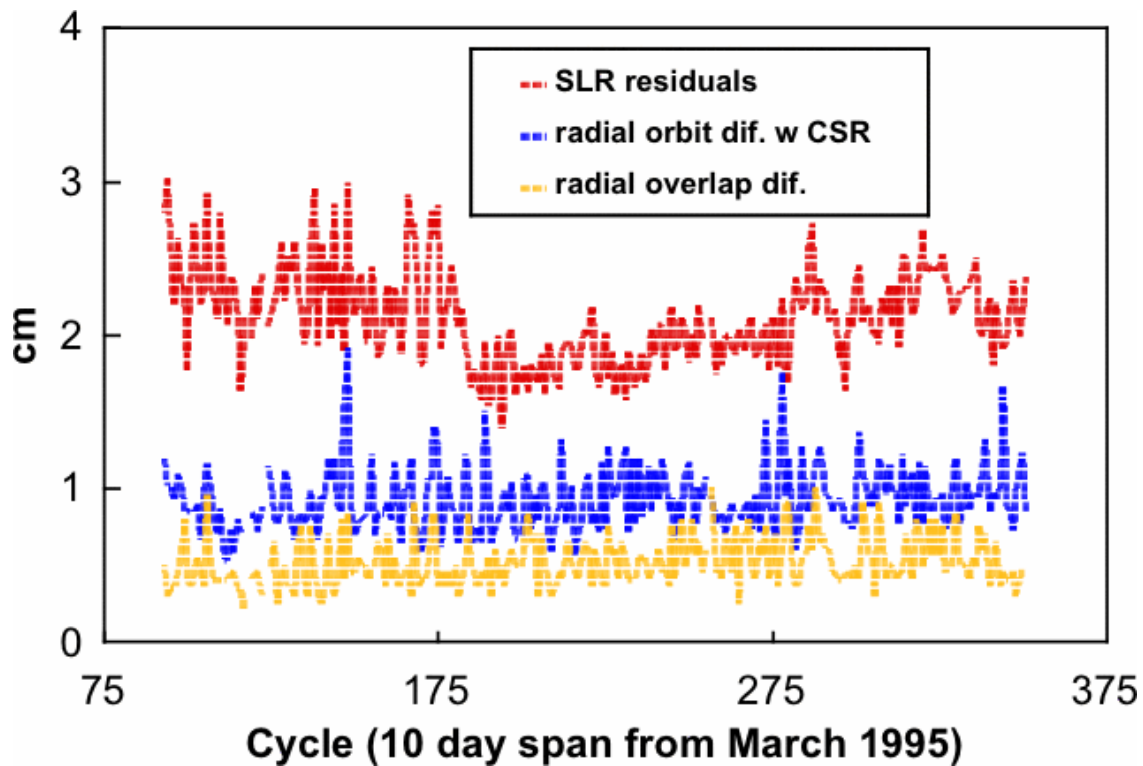


Figure 2. Orbit quality indices for Topex/Poseidon GDR orbits: SLR residual RMS, Radial orbit overlap differences with external analysis centers (University of Texas, Center for Space Research), and Radial Orbit Overlap Differences.

calibration and error projection on T/P, which encouraged the optimism for the 1992 orbit error estimates, and which laid the foundation for achieving and surpassing these orbit goals.

Gravity modeling improvements in support of the T/P mission began at GSFC in 1982. Error covariance studies performed at that time on the GEM-L2²⁴ gravity model revealed an expected error near 1 meter. Thus, a 10-fold improvement in gravity modeling was needed to meet the 13 cm radial accuracy goals of T/P. Over the ensuing decade, through a total redesign of the gravity model software, and through a recognition of the need for improvements in ancillary force models, several gravity models of progressively improved quality were developed using more tracking data, improved data analysis techniques and ancillary models. These efforts culminated in the pre-launch JGM-1 and post-launch JGM-2 gravity fields.²¹ The JGM-2 gravity field incorporated the first fifteen 10-day cycles of SLR and DORIS tracking data to tune the pre-launch JGM-1 model for the T/P orbit. The improvements in the gravity field allowed weaknesses to be observed and corrected in other POD models, and thus has driven improvements in all satellite force and measurement models. The initial 1st generation POE orbits, which were based on the JGM-2 gravity model, achieved the radial accuracy of 3-4 cm (see Table 2),⁶ far superior to the mission goal of 13 cm.

The availability of GPS data led to a new gravity model, JGM-3,¹⁴ which incorporated four cycles of T/P GPS data, as well as other satellite data. In 1995, the combined significant improvements in the gravity and tide models, reference frame definition and tracking station positions, and the surface force modeling led to the adaptation of new POD standards, and the 2nd Generation, JGM-3 based orbits. These orbits, based on SLR/DORIS tracking, achieved 2.0-2.5 cm accuracy,⁵ and were used for all of the GDRs up to cycle 359. In 2002, beginning with cycle 360, the ITRF2000 station coordinates²⁵ became the new standard replacing the CSR95 coordinates.²⁶ Following the demise of the DORIS receiver on November 1, 2004, orbits were computed using a combination of SLR and altimeter crossover data. Tests have shown the accuracy of the SLR/DORIS and SLR/Crossover orbits were comparable. The cycle 447-481 orbits were computed using SLR/altimeter crossover data. T/P cycle 481, in October 2005, was the last orbit computed for the T/P GDR.

The T/P GDR orbits were computed at the NASA Goddard Space Flight Center (GSFC) Space Geodesy Laboratory using the GEODYN orbit geodetic parameter and orbit estimation program.²⁷ GEODYN is a

least-squares batch filter that is used for orbit determination for analysis of terrestrial and interplanetary tracking data. The reduction of the tracking data were carried out within a controlled environment especially developed for orbit production, PODPS. The Precision Orbit Determination Production System (PODPS) was developed by the GSFC POD Group for greater automation and quality control in the production of precise orbits. Before export, each orbit was scrutinized using a battery of tests, including direct comparison with orbits from the CSR and CNES Analysis Centers, orbit overlaps, computation of altimeter crossovers, tests with high elevation SLR data and intercomparison of SLR-only, SLR/DORIS, and DORIS-only orbit solutions. Over the 13 years of the T/P mission life span, from September 25, 1992 to October 9, 2005, all POE orbits were delivered on schedule and with accuracies approaching 2 cm.

III.B. Satellite Surface Force Model

At the T/P altitude, the dominant surface force effects are due to solar, terrestrial, and thermal radiation pressure, with secondary effects due to atmospheric drag.²⁸ In order to meet the mission requirements, an exhaustive effort was undertaken to characterize the spacecraft shape, attitude, material properties and acceleration history. First finite element modeling of the surface forces was used to characterize the radiation forces.²² Since it was not feasible to implement a finite element thermal model in routine OD processing, a flat plate model approximation (macromodel) of the finite element model (micromodel) was developed.²³ From these investigations, the spacecraft was modeled in GEODYN as a set of eight flat plates arranged in the shape of a box and connected wing. Each plate is assigned properties including area, specular and diffuse reflectivity, emissivity, equilibrium temperature, and exponential decay times for panel cooling and heating after shadow entry or exit. These parameters approximate the aggregate composition and thermal behaviour of each side of the spacecraft (see Fig. 3).

The surface forces acting on each flat plate (solar radiation, albedo, thermal radiation, atmospheric drag) are computed independently, and then vectorially summed to produce the overall acceleration of the satellite center of mass. Prelaunch plate properties were derived from the finite element modeling and which were then tuned using SLR and DORIS data over cycles 1-48.⁵ It is believed the tuned model accounted for over 95% of the observed accelerations,²³ and that the residual nonconservative forces are largely accounted for through the adjustment of daily empirical accelerations.⁶ Nonetheless, surface force mismodeling remains a major contributor to the T/P orbit errors. It is possible to further reduce this error with the application of the reduced-dynamic technique, as will be discussed later in this paper.

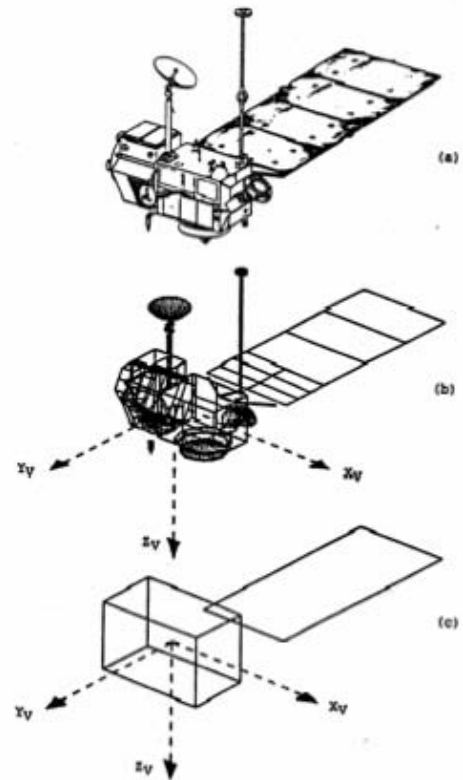


Figure 3. Representations of the Topex/Poseidon spacecraft: (a) full schematic; (b) micromodel; (c) macromodel

III.C. Gravity Field Model

Gravity field modeling has been the one most historically daunting challenges to improving orbit accuracy. Undulations in the Earth's gravity potential are complicated and mismodeling will result in perturbations to the satellite's position as a function of its path over the Earth's surface, even at the T/P altitude. The Earth's gravity field is modeled in spherical harmonics using normalized coefficients (\bar{C}_{lm} , \bar{S}_{lm}) with the equation²⁹

$$U = \frac{GM}{r} + \frac{GM}{r} \sum_{l=2}^{\infty} \sum_{m=0}^l \left(\frac{R_e}{r} \right)^l \bar{P}_{lm}(\sin \phi) [\bar{C}_{lm} \cos(m\lambda) + \bar{S}_{lm} \sin(m\lambda)] \quad (1)$$

where GM is the universal constant of gravitation times the mass of the Earth, l is the degree, m is the order, \bar{P}_{lm} are the fully normalized associated Legendre polynomials, R_e is the reference radius of the gravity model, ϕ is the latitude, and λ is the longitude.

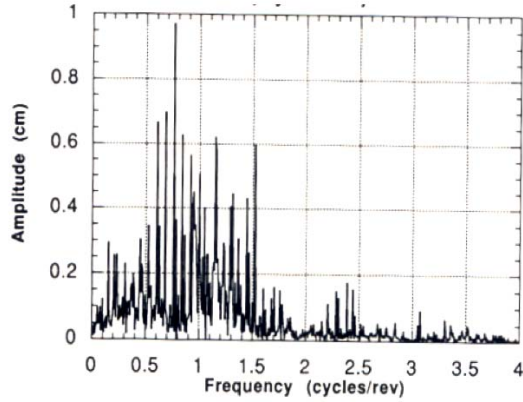


Figure 4. Amplitude spectrum of the radial orbit differences between the JGM-2, and JGM-2 clone orbits for T/P cycle 18.⁵

bations: (1) “ m -daily” or m cycles per day, where m is the order of the spherical harmonic expansion); (2) short period perturbations proportional to multiples of the orbital frequency and more than 1 cpr; (3) long period; (4) resonant perturbations with periods of 2-5 days.

We show in Fig. 4 the the complicated nature of the radial orbit error spectral characteristics due to gravity model error. In this case a T/P JGM-2 orbit is differenced with a JGM-2-clone orbit. The simulated gravity clone deviates from the original model by 1 fully correlated standard deviation and produces, through a simulation, realistic orbit error.²⁰ Orbit error arising from the gravity model can also be characterized as geographically correlated and geographically anti-correlated error.^{31,32} The geographically correlated error is the same for repeated ascending and descending overflights of the same region. The geographically anti-correlated (or variable) error changes sign for a satellite’s ascending versus descending track over a region. It should be noted the temporal variations of radial orbit error due to gravity can be mapped into such a spatial distribution.³¹ The geographically correlated orbit error directly affects the final sea surface altimeter measurement and is impossible to correct other than by improving the gravity model. The radial orbit error for the JGM-1, JGM-2, and JGM-3 gravity models is summarized in Table 3.

The availability of limited GPS data demonstrated that GPS tracking data and the reduced-dynamic approach could be used to achieve highly accurate orbits, considerably more accurate than the available 1st generation JGM-2 Precision Orbit Ephemeris (POE) orbits.¹³ Using the GPS orbits computed at JPL as a benchmark, differences with the POE orbits were used to characterize the error in the 1st generation POE orbits, and the reduction in error with the 2nd generation orbits using improved gravity and dynamic tide models. Due to limited availability of GPS tracking data, only portions of 37 T/P cycles (10 through 50) were available for this orbit comparison. Although the T/P GPS orbits computed at JPL were very accurate, they still contained a 3-4 cm bias in the Z-axis, an artifact that had to be removed for the best orbit comparison (see Fig. 5). In Fig. 5, we show the mean POE-GPS differences binned in $5^\circ \times 5^\circ$ blocks. Fig. 5a and Fig. 5b show the mean orbit differences for JGM-2 based POE’s and JGM-3 based POE’s respectively.⁵ The application of the JGM-3 gravity field dramatically

Using orbit perturbation theory it can be shown that the mismodeling of the gravitational field produces orbit errors at specific frequencies defined by the degree and order of the expansion in combination with the Keplerian orbit characteristics.²⁹ For the near-circular T/P orbit with a repeating ground track in which the argument of perigee is “frozen”, perturbation theory indicates that the gravity field produces a complicated error spectrum with the majority of the signal occurring at or near one-cycle-per-orbital revolution (1 cpr). This is also the dominant frequency for the nonconservative surface force model errors previously discussed. It has been shown the dominant 1 cpr orbit error signal can be effectively removed with the adjustment of empirical acceleration parameters derived from linear perturbation theory.³⁰ However the gravity field also gives rise to a more complicated error spectrum. The gravity field induces several classes of orbit perturbations:

Table 3. TOPEX/Poseidon Predicted Radial Orbit Error due to Gravity Model Errors

Predicted Error (cm)	JGM-1	JGM-2	JGM-3
total	3.4	2.2	0.9
geographically correlated	2.5	1.6	0.6

reduced the geographically correlated error. In Fig. 6a and Fig. 6b we show that the anti-correlated error associated with the JGM-2 gravity has also been reduced.⁵ The incorporation of GPS tracking of TOPEX in the JGM-3 gravity model dramatically improved the geopotential model for TOPEX, especially the $m = 1$ terms which contribute to the geographically correlated error.¹⁴

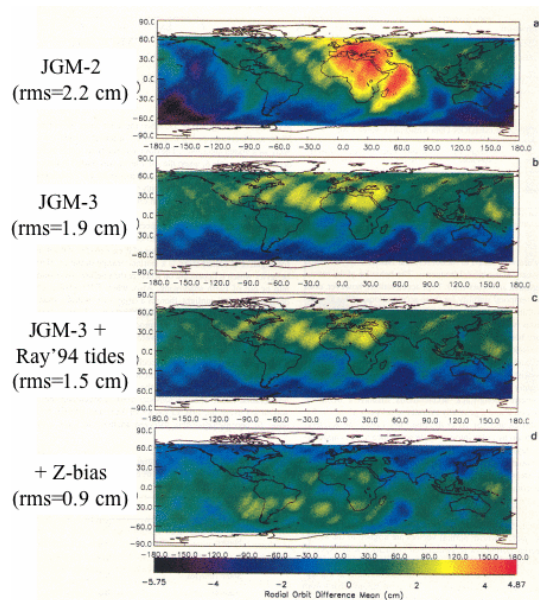


Figure 5. Illustration of the reduction of the geographically correlated orbit error through the application of new orbit models (from Ref. 5). Mean radial orbit differences between POE SLR/DORIS orbits, and GPS reduced-dynamic(RD) orbits in $5^\circ \times 5^\circ$ bins: (a) JGM-2 minus GPS RD; (b) JGM-3 minus GPS RD; (c) Second Generation POE's (JGM-3, Ray'94 tides) minus GPS-RD; (d) as in (c) but with Z bias removed.

III.D. Dynamic Ocean Tide Model

Table 4. Tidal Periods for TOPEX/Poseidon

Tidal Constituent	Period on the Earth's surface	T/P Aliasing Period (days)
M_2	12.421 hrs	62.1
S_2	12.000 hrs	58.7
O_1	1.076 days	45.7
K_1	23.935 hrs	173.2

only every 10 days. This sampling causes the T/P to observe tidal constituents having an alias period, usually much different than the period seen from the Earth's surface. A complete tidal cycle is sampled by T/P at a fixed point on the Earth's surface exactly over the alias period (see Table 4). Errors in modeling tides will manifest themselves in both the orbit (dynamic model) and ocean (geometric model) with similar spectra.³³ This is important for those trying to improve the ocean tide models with altimeter data. The tide models contain both adjusted (resonance) and unadjusted (short period) terms. Although the long period tidal terms in resonance with the T/P orbit produce the largest perturbations, they are well determined from satellite tracking data in geopotential solutions, and any residual error is absorbed in the adjustment of empirical acceleration parameters as part of the T/P POD strategy. It is predominantly the background tide model, which contains both omission and commission errors, and gives rise to short period orbit perturbations, which form the dominant error source in modeling tides. The ocean tidal model used in the 1st Generation POE was based on a combination of the Schwiderski oceanographic models.³⁴ The background

Tides modify the gravitational field, affecting the spacecraft orbit, and contribute to error in its determination. Tides also show up as part of the geometric signal in the altimeter measurements.² Tides are modeled as a combination of many constituent terms, but the largest tides on the ocean surface arise from solar and lunar effects, with the S_2 and M_2 constituents dominant in the semidiurnal band, and the O_1 and K_1 constituents dominant in the diurnal band. Although these tidal constituents have periods on the Earth's surface close to 12 and 24 hours respectively, a fixed point is sampled by T/P

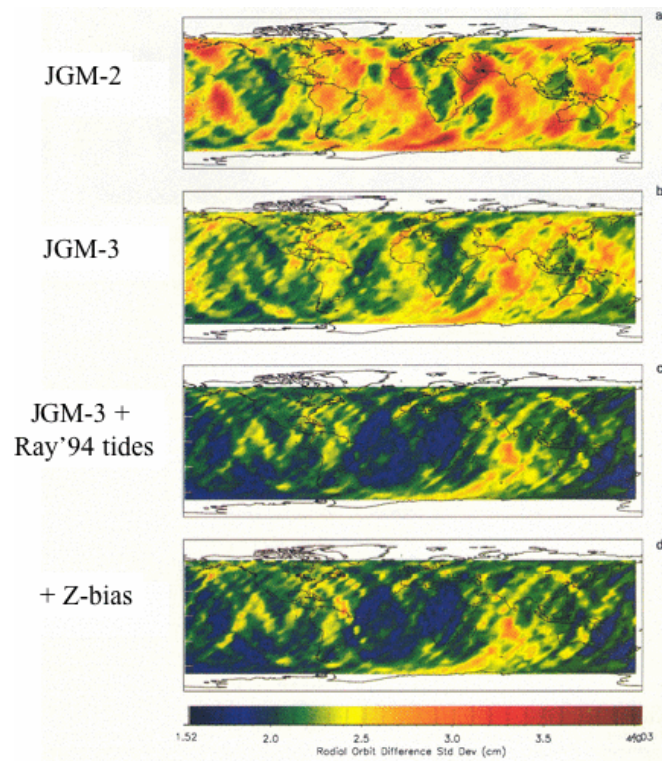


Figure 6. Illustration of the reduction of the anti-correlated orbit error through the application of new orbit models (from Ref. 5). Standard deviation about the radial mean, orbit differences between POE SLR/DORIS and GPS reduced-dynamic (RD) orbits: reduced-dynamics orbits in $5^\circ \times 5^\circ$ bins: (a) JGM-2 minus GPS RD; (b) JGM-3 minus GPS RD; (c) Second Generation POE's (JGM-3, Ray'94 tides) minus GPS RD; (d) as in (c) but with Z bias removed.

tide model that was developed for T/P using analytical orbit theory to evaluate the ocean tidal perturbations on the T/P orbit.³⁵ The 2nd Generation POE used a background tide model based on T/P altimeter data.³⁶

Using the GPS reduced-dynamic (RD) orbits as a benchmark, the change in the standard deviation (see Fig. 6b, 6c) indicates a dramatic reduction in the error variance upon switching from the Schwiderski-based to the T/P-based background tide models.

In order to evaluate the impact of orbit error on oceanographic analysis, one must also consider how the orbit error changes temporally at a fixed geographic location. This is accomplished through a spectral analysis of the POE-GPS orbit difference time series at each of the 14,400-longitude/latitude grid points in our orbit difference database. Each typical grid point in the database samples an orbit differences every 10 days over the 37 available cycles. We show in Fig. 7 presents a three dimensional map of the power spectra for each of the 14,400 geographically collocated points. Most of the strong signal lies at a 61-day period, with up to 2 to 3.5 cm amplitude peaks. A 45-day period is the second most powerful term.

The S_2 , M_2 , O_1 , and K_1 constituents account for more than 95% of the ocean tidal variance on the Earth's surface. The aliasing (or sampling) periods for these tides by the T/P orbit ground track (Table 4) closely match the 61-day and 45-day peaks seen in Fig. 7. The shortcoming in the Schwiderski-based background tide model used in the 1st Generation POE, dominated by M_2 , S_2 , and O_1 errors was believed to be responsible for the 61-day and 45-day periodicities seen in Fig. 7. Indeed, when a new T/P-based tide model³⁶ was used to compute new POE orbits, the orbit difference analysis showed a substantial decrease in power at all periods but most especially at the 61-day and 45-day periods (see Fig. 8).

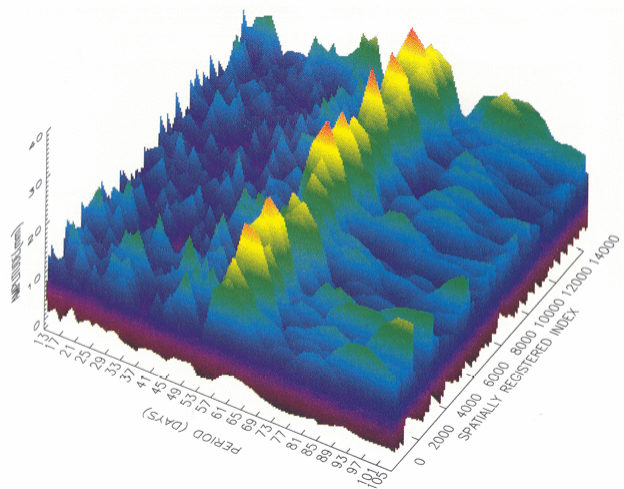


Figure 7. Amplitude spectrum of the 1st generation POE orbits minus the GPS reduced-dynamic orbit differences at fixed geographic points (from Ref. 5). This amplitude spectrum reveals that the 1st generation TP orbits contained error at specific tidal frequencies.

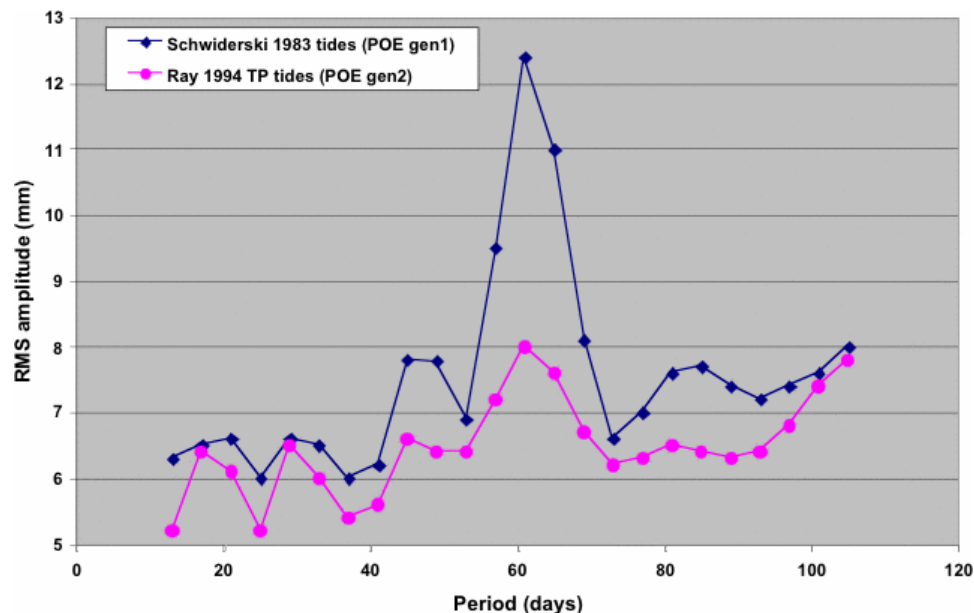


Figure 8. Geographic average RMS of the amplitude spectrum of the POE-GPS radial orbit differences for the 1st generation, and the 2nd generation POE orbits (from Ref. 5)

IV. Recent Significant POD Improvements

Orbits on the T/P mission Geophysical Data Records (GDRs) are accurate in the radial component at the 2 cm level.⁵ Recently it was demonstrated that it is possible to compute the radial component of the Jason orbits to better than 1 cm.^{16–18} Although this is accomplished largely on the strength of GPS tracking, the computation, verification and error characterization of such high accuracy orbits requires the reduction and analysis of all available tracking data: GPS, SLR, DORIS and altimeter ranging. The analysis of Jason-1's tracking data indicates that the history of T/P orbits could be considerably improved employing new solution strategies and models developed and tested on Jason-1. Current results from our efforts have shown considerable improvement in both the T/P and Jason-1 orbit accuracies. Significant improvements have been made with:

- Static and time-varying gravity.
- The ITRF2000 Terrestrial reference frame realization.²⁵
- The reduced-dynamic technique.
- The application of the IERS2003 standards.³⁷

The new POD reprocessing standards, still under development, will likely replace many of the GDR POD standard models as summarized in Table 5.

Table 5. NASA GSFC POD Processing Standards for TOPEX/Poseidon

T/P POD Model	GDR (1995)	New (2006)
Gravity (static)	JGM-3 (70x70)	GGM02C (120x120) ³⁸
Gravity (time-variable)	C20-dot, C21-dot, S21-dot	Same + 20x20 annual terms from GRACE
Atmospheric gravity	Not applied.	NCEP, 50x50@6hrs
Ocean tides	Ray94+GEMT3X ⁴¹	GOT00.2(20x20) ⁴²
Solid earth tides	$k_2=0.300$; $k_3=0.093$ + special modeling for FCN ⁴¹	IERS2003
Albedo/IR	Knocke et al., 1988 ⁴³	Same
Atmospheric drag	DTM ⁴⁴	MSIS86 ⁴⁵
Surface Forces	Tuned 12-panel model	Same
Data	SLR/DORIS	Same
Parameterization	Cd/8 hrs + opr along & cross- track/day; 10-day arc.	dynamic: Same reduced-dynamic possible
Station coordinates	CSR95L01 (cycle 1-359) ²⁶	ITRF2000, ²⁵ DPOD2000 ⁴⁰
Precession	IAU1976	IAU2000 ⁴⁶
Nutation	IAU1980+corrections	IAU2000 ⁴⁶

IV.A. Terrestrial Reference Frame Improvement

The GDR SLR/DORIS POE orbits had been computed using the CSR95L01 complement for the SLR stations and the CSR95D02 complement for the DORIS, and the corresponding Earth Orientation Parameter (EOP) CSR95L01- based series computed at the University of Texas, Center for Space Research (Richard Eanes, CSR memo May 1995).²⁶ The CSR95 SLR and DORIS position/velocity solution sets were determined at CSR using SLR and DORIS data through the spring of 1995. The CSR95 complements were highly accurate state-of-the-art realizations at the time, however for POD following 1995, the station positions had to be extrapolated and grew progressively worse in time, especially for DORIS.³⁹

In June 2002 PODPS replaced the CSR95 complements with ITRF2000,²⁵ and cycle 360 became the first GDR orbit based on the new station set. In the six months prior to the implementation, an extensive tests were completed at NASA GSFC and at UT/CSR to evaluate the SLR/DORIS the ITRF2000 complements for T/P POD. The original ITRF2000 SLR complement was considerably improved using refined

positions/velocities for many stations with updates provided by UT/CSR (Ries, J.C., personal communications, June 2002). An updated DORIS complement based on ITRF2000, DPOD2000, is used instead of the original ITRF2000 complement.⁴⁰

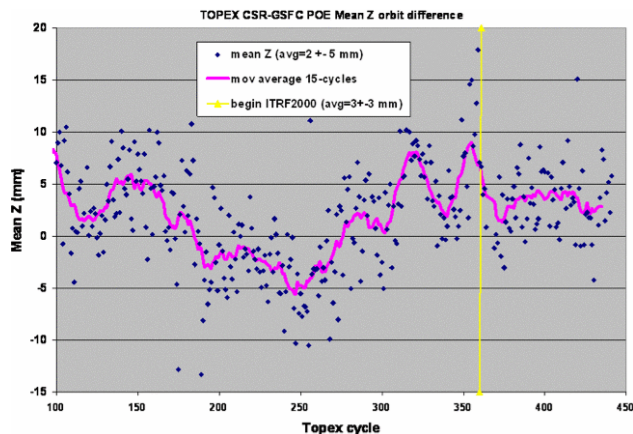


Figure 9. Orbit centering comparison in the Z coordinate, between TOPEX SLR/DORIS orbit solutions computed at NASA GSFC and at UT/CSR from 1995 to 2005.

The orbit provides the reference frame from which the altimeter measurements are made. Errors in the terrestrial reference frame can affect directly the stability and accuracy of the computed orbit reference frame. Orbit centering is a measure of orbit reference frame stability. Orbit differences in Z averaged over one cycle have been used as an index for orbit centering. The mean Z orbit differences between the T/P orbits computed at GSFC and CSR have typically shown a scatter of about 5 mm with the CSR95 coordinates. When both centers switched to ITRF2000, the scatter in Z dropped to 3 mm indicating a significant improvement in orbit centering and consistency using the new coordinates (see Fig. 9). A new reference frame solution, ITRF2005, will soon be released and is currently under evaluation.

IV.B. Orbit Estimation Strategy

The T/P POD philosophy has been to apply the best possible measurement and force models, and at the last step to apply the adjustment of empirical parameters to accommodate any residual error. Linear perturbation theory shows that most perturbing forces acting on the satellite, however complicated, will largely result in a 1 cpr perturbation of the satellite position; furthermore, the effect of the perturbing error forces can be largely removed with the adjustment of empirical acceleration parameters in the orbit solution.³⁰ The DORIS observations on TOPEX/Poseidon supply near-continuous coverage, typically 80% to 90% of the orbit. The dense tracking coverage afforded by DORIS, and even more so by GPS, allows more empirical parameters to be estimated. In the GDR SLR/DORIS second-generation POE orbits, we adjust empirical one cycle per rev (opr) accelerations once a day along-track and cross-track. Drag coefficients (c_d 's) are adjusted every eight hours over the 10-day solution arc. With this strategy of adjusting a minimum set of empirical accelerations, we still consider these SLR/DORIS GDR orbits to be dynamical solutions.

An alternative POD strategy, the reduced-dynamic, was applied by JPL to T/P orbit determination using the GPS data.¹³ The reduced-dynamic approach relies on the precision and density of the tracking data to accommodate orbit error having a more complicated spectrum, by estimating a constrained time series of empirical accelerations spaced over intervals much shorter than the orbit period. This technique will refine the orbit accuracy achieved in a dynamic solution with a given set of force models, but is not a substitute for improvements in those force models. Unfortunately, precise GPS T/P tracking was limited to only about a one-year span over which A/S had been turned off. With the near-continuous tracking of DORIS, a reduced-dynamic approach is still possible.⁴⁷ The approach in GEODYN is to adjust empirical accelerations along-track and cross-track to the orbit every quarter revolution. The empirical accelerations

Table 6. T/P Orbit Performance Over Cycles 344-364

Orbit (SLR/DORIS)	RMS residuals		
	DORIS (mm/s)	SLR (cm)	Crossover (cm)
GDR (2nd Generation POE) dynamic, JGM-3, CSR95 station set	0.467	2.522	5.618
JGM-3, ITRF2000 dynamic	0.467	2.024	5.564
JGM-3, ITRF2000 reduced-dynamic	0.465	1.979	5.545
GGM02C, ITRF2000 reduced-dynamic	0.464	1.873	5.496
GGM02C, ITRF2000+ time-variable gravity reduced-dynamic	0.464	1.787	5.456

in an orbit solution are tied together with exponential constraints, according to the relationship,^{16,48}

$$W_{jk} = \frac{1}{\sigma^2} e^{1 - \frac{|T_j - T_k|}{\tau}} \quad (2)$$

where W_{jk} is the weight for the constraint equation between two parameters, at time T_j , and time T_k ; σ is the process noise sigma (user input); τ is the correlation time. Using this approach, we can compute reduced-dynamic orbits for TOPEX/Poseidon using SLR and DORIS tracking data.^{16,19} SLR/DORIS POD improvement using the reduced-dynamic approach is evidenced by the reduction of variance in the residuals compared to the second-generation POE orbits, computed with JGM-3 (see Table 6). In addition, the reduced-dynamic approach can better accommodate mismodeled forces. As an illustration, we show the recovered along-track accelerations for cycle 314 in March 2001 in Fig. 10. During this time, T/P experienced a sudden, severe, and short increase in atmospheric drag. The standard approach in the 2nd Generation POE's did not resolve these sharp changes. The application of the reduced-dynamic technique not only significantly improved the orbit, but the correlation with the K_p magnetic index indicates the recovered accelerations represent actual mismodeled forces.

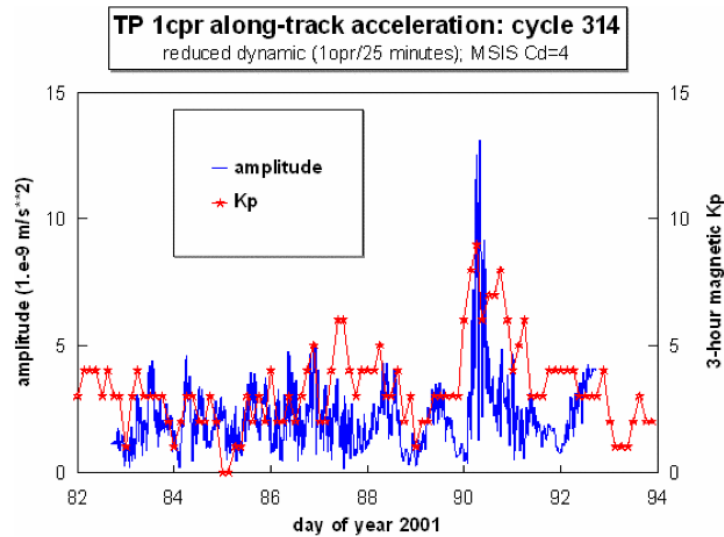


Figure 10. Recovered along-track accelerations for T/P cycle 314 computed using the reduced-dynamic technique. In this example, the accelerations can better track the mismodeled forces.

IV.C. Static and Time-Varying Gravity Field Models

The GRACE mission has permitted the development of new models of the Earth's gravity field that are significant improvements over earlier models.³⁸ We have tested the application of a new GRACE model, GGM02C, on T/P and find that it substantially improves both the RMS of fit to the SLR data and to the altimeter crossovers (see Table 6). In addition to improvements in the static gravity field, we represent the Earth's time-variable gravity (TVG) using two models: (1) Spherical harmonic coefficients to degree and order 50 every six hours representing atmospheric mass variations derived from NCEP pressure data,^{49,50} and (2) annual variations in the gravity field through degree and order 20 using results from the GRACE mission.^{51,52} The atmospheric gravity model makes the inverse barometer assumption over the oceans. In Table 6 we show the effect of applying these models to the T/P cycles 344-364. The reduction in the variance of the crossover residuals from 31.562 cm² to 29.768 cm² suggests that the orbit error has been reduced to approximately 1.5 cm from the 2.5 cm level of the 2nd Generation T/P POE's. It is also interesting that we see these improvements in the T/P orbits, even with the application of the reduced-dynamic technique.

V. Summary

We have discussed in this paper the history of Topex/Poseidon precision orbit determination, and how we were able to achieve routine and reliable delivery of orbits with a radial accuracy of 2.5 cm. This success was due to the precision of the complementary tracking systems, SLR, DORIS, and GPS that were carried by the spacecraft. The precision tracking systems permitted and even required continuous refinement in the force and measurement models that underpinned the POD calculations. The tracking systems in combination with dedicated analysis by the NASA GSFC, NASA/JPL, UT/CSR, and the CNES analysis centers working in both friendly competition and collaboration were keys to the mission success. When we see a time series of T/P altimeter-derived ocean heights that show the evolution of El Niño across the Pacific Basin, or evaluate a calculation of the change in mean sea level, we must not forget how the precision orbit determination success is absolutely vital to those results. With the experience obtained on Jason-1, and the development of refined models of the Earth's gravity field from missions such as GRACE and the elaboration of new realizations of the terrestrial reference frame, we can expect further improvements to the T/P radial OD accuracy from the 2.5 cm level of the 2nd Generation POE's to the 1.5 cm level or better.

Acknowledgments

The success of the TOPEX/Poseidon mission is due in no small part to the dedication of the teams that have operated and maintained the SLR, DORIS and GPS tracking networks from 1993 to 2005. We acknowledge the many satellite laser ranging stations around the world who made such special efforts to track T/P on a routine basis. In addition, we cannot fail to acknowledge the efforts of the Institut Géographique Nationale and the CNES to maintain and upgrade the DORIS network since 1993. We acknowledge the project teams at the CNES and at JPL who managed the spacecraft and provided the mission data and support that were essential to the POD computations. We note the special roles played directly and indirectly by the University of Colorado and the Ohio State University in TOPEX gravity model development and error calibration and validation. Finally, we must acknowledge colleagues no longer with us who were vital members of the Topex/POD team, including James G. Marsh, Francis J. Lerch, Girish B. Patel, and Edward J. Christensen.

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